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Why Counterpossibles are Non-Trivial

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Subjunctive conditionals with impossible antecedents (or counterpossibles) are standardly treated as vacuously true, the lore being that if an impossibility were to obtain, anything would follow. Daniel Nolan (1997) and others have argued that there are several good reasons to steer clear of the standard reading. In this note we provide further reasons.

I. Non-Trivial Counterpossibles

On Lewis' account, a subjunctive of the form 'if it were the case that *p*, it would be the case that *q*' (represented as '$p \mathbin{\Box\!\!\rightarrow} q$') is to be given the following rough meta-linguistic truth-conditions[1]:

> '$p \mathbin{\Box\!\!\rightarrow} q$' is true at a world of utterance *w* iff every closest p-world is a q-world.

The closeness of worlds is a highly context sensitive matter, since understood, roughly, as *relevant* similarity. The worlds most relevantly similar to *w* will be the ones that share with *w* the most background facts held fixed in the conversational context.

Lewis' account of subjunctives entails that a subjunctive, $p \mathbin{\Box\!\!\rightarrow} q$, with an impossible antecedent, p, is vacuously true. For if there are no *p*-worlds, then vacuously all the closest *p*-worlds are *q*-worlds. Accordingly, counterpossibles are trivially true: an impossible antecedent counterfactually implies the truth of anything you like, and for this reason, the truth value of a counterpossible does not depend in any way on the truth value of the consequent.

[1] Meta-linguistic truth-conditions are to be distinguished from object-language truth-conditions (or the proposition expressed).

Lewis' account has much to recommend it. It makes good predictions as regards most run of the mill subjunctives. As Daniel Nolan (1997) has argued, however, this treatment fails to accommodate our intuition that some counterpossibles are non-trivially true and some are non-trivially false.[2] Consider, for instance (Cf. Nolan 1997: 544):

(1) If Hobbes had (secretly) squared the circle, sick children in the mountains of South America at the time would not have cared.

(2) If Hobbes had (secretly) squared the circle, then Hobbes would not have squared the circle.

There is a strong intuition to the effect that (1) is non-trivially true, and (2) is non-trivially false. The general reason being that the consequent of each seems relevant to the truth value of the whole. But on Lewis' account, both are trivially and uninformatively true.

To account for counterpossibles non-trivially, Nolan proposes to introduce impossible worlds. Since impossibilities entail everything in classical logic and we wish to avoid triviality, an impossible worlds account of counterpossibles would seem to require us to weaken our consequence relation in some uniform way. For instance, one might, for the general case, evaluate a counterpossible with some favored paraconsistent logic (i.e., a logic in wish contradictions do entail everything). But, says Nolan, a uniform weakening of the consequence relation is a bad idea. We agree. Consider: 'if classical logic were incorrect, paraconsistent logic would be correct'. That seems false, but comes out true if we simply weaken the consequence relation to a favored paraconsistent logic. It seems false because some relevantly close non-classical worlds are not paraconsistent, but rather are intuitionistic.[3] But it comes out true on the

[2] See also Kment (2006a, 2006b).

[3] The approach here is not to be thought of as combined with the Nolan approach of treating impossible worlds as non-deductively closed (see next paragraph). Once we treat impossible worlds as non-deductively closed, we no longer have a need for uniformly weakening the consequence relation.

approach that weakens the consequence relation to some paraconsistent logic, because---on such a weakening---paraconsistent logic is correct in every paraconsistently possible world, and so, is correct in the closest paraconsistently possible worlds where classical logic is incorrect. Another example is this: in the context of a classical logic course a student would be correct in pointing out that if a contradiction were true then everything would follow. But with a uniform paraconsistent consequence relation for counterpossibles, this student should be censored.

The objection to uniformly weakening the consequence relation doesn't turn on which relation we used in the examples. The examples could easily be modified mutatis mutandis to repudiate one's favored weakening of classical logic. It would seem then that any uniform weakening of the consequence relation will fail by itself to capture all reasonable intuitions about counterpossibles.

Nolan suggests that we treat impossible worlds as logically anomalous. With the exception of the absurd world – the world in which every proposition is true – impossible worlds are maximal non-deductively closed sets of propositions or sentences.[4] The absurd world is both maximal and deductively closed. A set S is maximal iff for any proposition (sentence) *p*, either *p* or not-*p* (or both) is a member of S. A set S is not deductively closed iff not every consequence of the propositions (sentences) in S is a member of S. Accordingly, *p* is true at an impossible world *w* iff *p* is a member of *w*.

On Nolan's account, impossible worlds are ordered relative to how similar they are to the actual world, given certain background facts. As Nolan puts it,

> some impossible worlds are more similar, in relevant respects, to our actual world than others. The "explosion" world – the impossible world where every proposition is true – is very dissimilar from our own. Indeed it seems to be one

[4] Nolan leaves open whether worlds are concrete or abstract. However, since inconsistent beings are more bizarre than inconsistent sets, we will treat worlds as abstract. Moreover, in the appendix Nolan seems to allow for non-maximal worlds. For simplicity's sake, we shall here treat worlds as maximal. In forthcoming work we develop the account in further detail.

> of the most absurd situations conceivable. On the other hand, the world which is otherwise exactly like ours, except that Hobbes succeeded in his ambition in squaring the circle (but kept it a secret), is far less dissimilar (1997: 544).

Generally, impossible worlds with just a few violations of the physical/metaphysical/logical laws, and otherwise as similar as can be to the actual world, are closer to the actual world than worlds with a large number of violations.

II. An objection to Non-Trivial Counterpossibles

Nolan's view of counterpossibles appears to have some degree of initial plausibility. However, Timothy Williamson argues that a non-vacuous reading of counterpossibles runs into severe difficulties (see Williamson 2006: Lecture #3). Williamson considers the intuitive pull of counterexamples such as:

> (3) If 5 + 7 were 13, then 5 + 6 would be 12.

The idea is that (3) is alleged to be non-vacuously true. But Williamson argues that if we play along and think through the example, trouble emerges. Accepting (3) as non-vacuously true, 5+5 would be 11, and 5+4 would be 10, and … , and 5+(-5) would be 1. Accordingly, 0 would be 1. Holding the context fixed, it follows that

> (4) If the number of answers I gave (to a given question) were 0, then the number of answers I gave would be 1,

which is plainly false.

However, the discussion is not convincing. The reading of (4) which is plainly false is one where the closest antecedent-worlds are *possible* worlds. For the closest possible worlds in which the number of answers I gave is 0, are not worlds where the number of answers I gave is 1. Instead, if we play along with an impossible worlds account, the following counterpossible comes out true:

> (5) if 0 were 1 and the number of answers I gave were 0, then the number of answers I gave would be 1.

This should sound (and should be treated as) non-vacuously true to one countenancing (non-deductively closed) impossible worlds.

Our reply to Williamson will be unsatisfying to one who thinks that we wrongly assumed that Williamson held the context fixed across the inference from (3) to (4). But if the context is not held fixed then (4) doesn't follow from (3), and Williamson loses his objection to the non-vacuously true reading of (3). Either way, context held fixed or not, Williamson's objection fails to convince.

We do agree with Williamson that one should not rest content with a mere appeal to our pre-theoretical intuitions about counterpossibles. But there are other compelling pieces of evidence for taking subjunctives with impossible antecedents to be non-trivial. One is this: counterpossibles are frequently employed by philosophers to develop or refute their opponents' positions. So, if all counterpossibles were trivially true, much of philosophy would be less substantial than it is.[5] Philosophical reasoning often involves non-trivial counterpossible reasoning. Even Williamson's reasoning in places tacitly presupposes non-trivial counterpossibles. Right after expressing the view that counterpossibles are vacuously true, for instance, he considers the view that they are all vacuously false. He writes,

> If all counterpossibles were false, $\Diamond A$ would be equivalent to $A \Box\!\!\rightarrow A$, for the latter would still be true whenever A was possible; correspondingly, $\Box A$ would be equivalent to the dual $\neg(\neg A \Box\!\!\rightarrow \neg A)$. And one could carry out the programme of section 3 using the new equivalences. (2006)

[5] A version of the point is made in Nolan (1997: 539-540).

The first sentence of this quote is a counterpossible (on the assumption that, necessarily, all counterpossibles are true). So if all counterpossibles are in fact true, what Williamson just said is vacuously true. But so is its complement[6]---viz.,

> If all counterpossibles were false, ◊A would *not* be equivalent to A $\Box\!\!\rightarrow$ A …

So why didn't Williamson make his point with the complement? Because it wouldn't have been informative. That is, opting for the vacuously true reading, the consequent would not have contributed to the truth-value of Williamson's claim. Williamson presumably does not intend the above paragraph to be uninformative and vacuous. The vacuity treatment of counterpossibles that he proposes is then not the view he intends his readers to employ when evaluating his above claim.

Since Williamson in the above quote, along with many other philosophers in our discipline, make deep and philosophically informative claims with their counterpossible assertions, a non-vacuous reading of these particular constructions is in order. The general lesson is that if our counterpossible philosophical claims are non-trivial and informative, and we presuppose they are, then it would be wrong to treat them without exception as vacuously true.

There are other reasons for allowing non-trivial counterpossibles. As Kit Fine (1994) has argued,[7] essential properties are not reducible to necessary properties. While Kripke's wooden table, Tabby, is necessarily a member of the set {Tabby}, it is not essential to Tabby that it be a member of that set. Nor is it essential to Tabby that seven is prime or that it be such that either it's raining or it's not raining. The properties: being a member of the set {Tabby}, being such that seven is prime, and being such that either it's raining or it's not seem irrelevant to the question of what it is to be Tabby.

[6] We label these formulas 'complements' for ease of exposition. Strictly speaking $p \Box\!\!\rightarrow \sim q$ is the *complement* of $p \Box\!\!\rightarrow q$ iff counterfactual excluded middle, $(p \Box\!\!\rightarrow q) \vee (p \Box\!\!\rightarrow \sim q)$, is valid. For reasons given in Lewis (1979), we do not take counterfactual excluded middle to be valid.
[7] See also Kment (2006b: 244)

It is tempting to offer the following counterfactual explanation: if there hadn't been sets, Tabby might still have existed; if seven hadn't been prime, Tabby might still have existed; and if there had been counterexamples to '*p* or not-*p*', Tabby might still have existed. But this sort of explanation requires, for its non-triviality and informativeness, that counterpossibles be non-trivial and informative. At the closest impossible worlds at which there are no sets or numbers (perhaps because there are no abstract entities), Tabby might still exist. At the closest impossible worlds where there are some true contradictions, Tabby might still exist. But following Kripke's (1980), at the closest worlds where there is no wood, Tabby does not exist. Non-trivial counterpossibles thus make a modal analysis of essences possible. *a* is essentially *F* iff if nothing had been *F*, then *a* would not have existed.

We conclude that there are several good reasons for embracing a treatment of counterpossibles as non-vacuous: besides preserving the familiar pre-theoretic intuitions, it protects against the triviality of philosophy, and facilitates a modal treatment of essential properties.

References

Fine, K. 1994. "Essence and Modality", *Philosophical Perspectives* 8: 1-16.

Kment, B. 2006a. "Counterfactuals and Explanation", Mind 115: 261-310.

2006b. "Counterfactuals and the Analysis of Necessity", *Philosophical Perspectives* 20, ed. John Hawthorne, 237-302.

Kripke, S. 1980. *Naming and Necessity*, Cambridge, MA: Harvard University Press.

Lewis, D. 1973. *Counterfactuals*, Oxford: Blackwell.

Nolan, D. 1997. "Impossible Worlds: A Modest Approach", *Notre Dame Journal for Formal Logic* 38: 325-527.

Williamson, T. 2006. *The Philosophy of Philosophy*, Carl G. Hempel Lecture, Lecture #3, Princeton.