

On Keeping Blue Swans and Unknowable Facts at Bay. A Case Study on Fitch's Paradox

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I. Introduction

What has come to be called the *knowability paradox* was first published by Frederic Fitch as Theorem 5.¹ It is equivalent to the claim that if every truth is knowable, then every truth will be known:

$$(T5) \quad \varphi \rightarrow \diamond K\varphi \dashv\vdash \varphi \rightarrow K\varphi$$

where \diamond is possibility, and 'K φ ' is to be read as φ is known by someone at some time. Let us call the premise the *knowability principle* and the conclusion *near-omniscience*.² Here is a way of formulating Fitch's proof of (T5). Suppose the knowability principle is true. Then the following instance of it is true: $(p \ \& \ \sim Kp) \rightarrow \diamond K(p \ \& \ \sim Kp)$. But the consequent is false, it is not possible to know $p \ \& \ \sim Kp$. That is because the supposition that it is known is provably inconsistent.³ The inconsistency requires us to deny the possibility of the supposition, yielding $\sim \diamond K(p \ \& \ \sim Kp)$.

This, together with the above instance of the knowability principle, entails $\sim(p \ \& \ \sim Kp)$, which is (classically) equivalent to $p \rightarrow Kp$. Since p occurs in none of our undischarged assumptions, we may generalize to get near-omniscience, $\varphi \rightarrow K\varphi$. QED.

¹ F. Fitch, "A Logical Analysis of Some Value Concepts" *The Journal of Symbolic Logic* 28 (1963), 135-142, p. 139.

² I used to call the conclusion *omniscience*. But, of course, $\varphi \rightarrow K\varphi$ does not entail omniscience, i.e., that there is someone who knows all truths ($\exists \forall$), but only the weaker claim that all truths are known by someone ($\forall \exists$). Thanks to Michael Hand for suggesting that I re-name it.

³ If $p \ \& \ \sim Kp$ is known, then it is true, giving $p \ \& \ \sim Kp$, and so $\sim Kp$. Also, if $p \ \& \ \sim Kp$ is known, then the left conjunct is known, giving Kp .

(T5) is today considered by many to be a paradox for a number of related reasons, among others, that it threatens to show that the very thesis that is thought to discriminate a mature semantic anti-realism from a naïve idealism entails that very idealism. A number of strategies have been developed to avert the paradox, and several of them have provoked significant and interesting debate.⁴ What has rarely, if ever, been noted, however, is that Fitch-like paradoxes threaten to undermine not only semantic anti-realism, but also potentially a number of other anti-realisms with superficial resemblance to semantic anti-realism. One form of anti-realism that is troubled by a Fitch-like paradox is what has come to be called *strong modal fictionalism*.⁵ Strong modal fictionalists hold that possible world talk, like literary fiction, is literally false. Nonetheless, they think the fiction provides the resources for an analysis of modal claims.

In this note I develop a Fitch-like paradox for strong modal fictionalism. I argue that the most promising strategy to avoid paradox is to reject the claim that modal claims are to be analyzed in terms of the contents of the fiction of possible worlds. It is hoped that by looking at the parallel case of modal fictionalism light can be shed on the threat posed by Fitch's paradox to semantic anti-realism.

II. Modal Fictionalism

Since Gideon Rosen's centerpiece of 1990, modal fictionalism has been taken seriously by many as a way to employ the resources of possible-world semantics without any of the usual ontological commitments. Modal fictionalism holds that possible world talk is to be treated on a par with talk of fictional objects, such as Sherlock Holmes. Like talk of fictional objects, possible

⁴ For an overview of the literature, see B. Brogaard and J. Salerno, (2004) "Fitch's Paradox of Knowability" in E. N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*, <http://plato.stanford.edu/archives/sum2004/entries/fitch-paradox/>.

⁵ This position originates in G. Rosen, "Modal Fictionalism", *Mind* 99 (1990), 327-54. Rosen also considers an alternative position which he calls '*timid modal realism*'. As we will see, this position is salvageable from paradox (or at least, salvageable from this paradox). For a rich overview of the literature, see D. Nolan "Modal Fictionalism" in E. N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*, <http://plato.stanford.edu/archives/sum2002/entries/fitch-paradox/>.

world talk is literally false (or untrue).⁶ It is literally false that there is a brilliant detective at 221b Baker Street. Likewise, modal fictionalists say, it is literally false that there are merely possible worlds, and merely possible objects. Thus, while there might have been blue swans, there is no possible world where there are blue swans.

What distinguishes fictionalists from eliminativists is that fictionalists hold that modal claims can be explicated in terms of possible worlds, as long as quantification over possible worlds occurs within the scope of an implicit story prefix (e.g., ‘according to the possible worlds fiction’). Quantification within the scope of a story prefix is, familiarly, not existentially committing.

The content of the possible worlds fiction is standardly taken to be David Lewis’ theory of possible worlds,⁷ including an encyclopedia, that is, a list of all literally true non-modal propositions. Following Rosen, p is a non-modal proposition just in case ‘it contains no modal vocabulary and entails neither the existence nor the non-existence of things outside our universe’.⁸

Where p is a sentence of quantified modal logic, p^* is a translation of p into the language of possible worlds, and W is a story prefix which reads: *according to the Lewis story*, Rosen’s formulation of modal fictionalism may be given as follows:

$$\text{(Fic)} \quad p \leftrightarrow Wp^*$$

(Fic) says that a modal claim will be true iff its translation into the language of possible worlds is true in the Lewis story. So, for example, ‘there might have been blue swans’ is true iff, according to the Lewis story, there is a possible world where there are blue swans. Likewise, ‘there are no blue swans’ is true iff according to the Lewis story, in the *actual* world, there are no blue swans.

⁶ More carefully: possible world talk that incurs a commitment to merely possible worlds or merely possible individuals is literally false (or untrue).

⁷ D. Lewis, “Counterpart Theory and Quantified Modal Logic”, *The Journal of Philosophy* 65 (1968), 113-126, reprinted in Lewis, *Philosophical Papers Vol 1* (Oxford: Oxford University Press, 1983).

⁸ “Modal Fictionalism”, p. 335.

A problem for modal fictionalism was noticed independently by Rosen and Stuart Brock.⁹ In a nutshell, it is that since in the Lewis story it is true at each world that there exists a plurality of worlds, we can derive, by (Fic), that necessarily there is a plurality of worlds. Since necessity entails truths, there in fact exists a plurality of worlds, which is not something the fictionalist should tolerate.

However, the objection has been shown to be unproblematic if careful attention is paid to the translation scheme offered by Lewis in 1968 for translating sentences in the language of quantified modal logic into sentences in the language of counterpart theory.¹⁰ In the 1968 translation scheme, the sentences of quantified modal logic translate into sentences that quantify over worlds and their parts. Thus, where ‘U’ means *world*, and I is the existing-wholly-in (or parthood) relation, a modal sentence of the form:

$$\Diamond \exists x Fx$$

translates as:

$$\exists x \exists y (Uy \ \& \ Ixy \ \& \ Fx)$$

This says that there is a world of which something that is *F* is part. Assuming the letter of the 1968 translation scheme, ‘necessarily, there is a plurality of worlds’ translates as:

$$\forall x (Ux \rightarrow \exists y \exists z (Iyx \ \& \ Izx \ \& \ Uy \ \& \ Uz \ \& \ y \neq z))$$

This says that all worlds have at least two worlds as parts. But this is *false*, according to the Lewis story, since no world has any other world as a part. Hence, the objection fails.

⁹ Rosen, “A Problem for Fictionalism About Possible Worlds”, *Analysis* 53 (1993), 71-81, and S. Brock, “Modal Fictionalism: A Response to Rosen”, *Mind* 102 (1993), 147-50. For a much more substantial treatment of the history of the debate and alternative solutions to problems, see Nolan “Modal Fictionalism”.

¹⁰ See H. Noonan, “In Defense of the Letter of Fictionalism”, *Analysis* 54 (1994), 133-39. The 1968 translation scheme is the one found in Lewis, “Counterpart Theory”. For discussion, see also Nolan “Modal Fictionalism”.

However, too careful attention to the 1968 scheme leads to Hale’s dilemma.¹¹ The fictionalist cannot say that modal realism is possible. For assuming the letter of the 1968 translation scheme ‘it is possible that there is a plurality of worlds’ translates as:

$$\exists x(Ux \ \& \ \exists y\exists z(Iyx \ \& \ Izx \ \& \ Uy \ \& \ Uz \ \& \ y \neq z))$$

This says that some world has at least two worlds as parts. But this is false, according to the Lewis story. Nor can he say that modal realism is impossible. For, Hale argues, the fictionalist must provide some analysis of the story prefix ‘according to the Lewis, story’. Most plausibly this explication will involve a non-material conditional of the form: if modal realism is true, then p . But if it is necessarily false that there is a plurality of worlds, then this conditional will be trivial; ‘according to the Lewis story, p ’ will be true for any p . Hence, the fictionalist will be committed to the truth of any modal claim.¹² The same problem arises if the fictionalist holds that the story prefix is primitive. The fictionalist ought to accept some closure principle of the form, $Wp, p \Rightarrow q \vdash Wq$. But where p is impossible (e.g. ‘there is a plurality of worlds’), p entails any claim. So, Wq obtains for any q . If q is ‘there is no world where there are blue swans’, we can derive (by Fic), the far-fetched claim ‘ $\sim\Diamond$ (there are blue swans)’.

John Divers has offered the following resolution of Hale’s dilemma.¹³ When the realist assents to ‘there is a plurality of worlds’ his intention is that the ‘conventional world-restriction of

¹¹ B. Hale, “Modal Fictionalism: A Simple Dilemma”, *Analysis* 55 (1995), 63-67. I am simplifying the first horn of the dilemma.

¹² The suggestion is Rosen’s [“Modal Fictionalism Fixed”, *Analysis* 55 (1995), 67-73]. Rosen also suggests that the fictionalist could block Hale’s dilemma by, for example, rejecting “classical” semantics for non-material conditionals, or allowing truth-values to statement involving the ‘world’ predicate only when the predicate occurs within the scope of the prefix. Hale [“A Desperate Fix”, *Analysis* 55 (1995), 74-81] thinks these suggestions are desperate. Disallowing truth-values to the statements involving “free” occurrences of the ‘world’ predicate would leave claims like ‘there is exactly one world, namely the actual’ without a truth-value. Rejecting the claim that counter-possible conditionals are all trivially true is plausible for conditionals with metaphysically impossible but logically possible antecedents. But Rosen’s suggestion is the more radical one that even counter-possible conditionals with logically impossible antecedents may fail to be trivially true. For further problems with these strategies see J. Divers and J. Hagen, “The Modal Fictionalist Predicament”, forthcoming in F. MacBride (ed.), *Identity and Modality* (Oxford: Clarendon Press).

¹² Divers, “A Modal Fictionalist Result”, *Nous* 33 (1999), 317-46.

¹³ Divers, “A Modal Fictionalist Result”.

quantification should not apply'.¹⁴ But, Divers argues, if restricted to alethic modality, the T axiom, $p \vdash \diamond p$, 'approaches the status of analyticity'.¹⁵ Hence, the realist must assent to 'it is possible that there is plurality of worlds'. In such cases, Divers argues, the operand modality must be read as redundant. That is, where p is unrestricted, the realist must assent to $\diamond p \dashv\vdash p$.¹⁶ Where the realist translates the unrestricted possibility claim $\diamond p$ as p , the fictionalist will thus do well to translate it as Wp . That is, the fictionalist can assent to the following instance of (Fic):

If p is read as an unrestricted possibility claim, then $(\diamond p \leftrightarrow Wp)$

Since the fictionalist is thus able to assent to the *contingent* falsehood of modal realism, Hale's dilemma is blocked.

There is, however, a different path to disaster even assuming Divers' counterpart-theoretic translation principles for possibility claims.

III. A Fitch-like Paradox

A collapse ensues owing to a Fitch-like proof of the following theorem (where p is an unrestricted locution):¹⁷

$$(T2) \quad p \rightarrow Wp^* \vdash \Box(Wp \rightarrow p)$$

Like the logic of Fitch's proof, the logic of the Fitch-like proof of T2 is modest: minimal, normal modal logic, and three intuitive story-prefix principles:

¹⁴ "A Modal Fictionalist Result", p. 323, and also Divers, "A Genuine Realist Theory of Advanced Modalising", *Mind* 108 (1999), 217-39.

¹⁵ "A Genuine Realist Theory", p. 218.

¹⁶ $\Box p \dashv\vdash p$ trivially follows. S5 and $\diamond p \vdash p$ yields the trivial system (Tr), in which no significant modal distinctions can be drawn.

¹⁷ The star is not required on the left-hand side of the (T2). Starred statements are translations of the statements of quantified modal logic into statements of counterpart theory. But given Divers translation scheme, unrestricted locutions of the form ' $\diamond p$ ' translate as ' p '. So, the star drops off. Thanks to an anonymous referee here.

- (A) $W(p \ \& \ q) \vdash Wp \ \& \ Wq$
- (B) $WWp \vdash Wp$
- (C) $Wp \vdash \sim W\sim p$
- (D) $\sim\Diamond p \vdash \Box\sim p$

(A) is a distributivity principle that says that if in the Lewis story, p and q , then in the Lewis story, p , and in the Lewis story, q .¹⁸ (B) says that if the Lewis story states that according to the Lewis story p , then the Lewis story states that p . (C) states: if in the Lewis story, p , then it is not the case that in the Lewis story, not- p . (D) is the inference of a necessary falsehood from an impossibility.

If these resources are not already believable, we can say this. (A) is entailed by the uncontroversial assumption that conjunction in the Lewis story is classical. Denying (B) would yield the implausible result that the Lewis story may deny p yet assert about itself that it holds that p (certainly, Lewis would have disapproved of such a theory).¹⁹ (C) follows from the uncontroversial assumption that modal realism is classically consistent. (D) follows from the duality of the modal operators.

The proof employs a theorem derived from these resources, Theorem T---viz., for any p , it is not the case that according to the Lewis story, both not- p , and according to the Lewis story, p .

$$(T1) \ \sim W(\sim p \ \& \ Wp)$$

¹⁸ Of course, to avoid obvious counterexamples, we will ultimately need an account of tense operators occurring within the scope of a story prefix. Thanks to David Jehle here.

¹⁹ Notice, further, that (B) is validated on Divers' ["A Modal Fictionalist Result"] explication of the story prefix: $\Box((Wp) \leftrightarrow \Box(\text{the Lewis story} \rightarrow p))$, where the box is primitive. For if $\Box(\text{the Lewis story} \rightarrow \Box(\text{the Lewis story} \rightarrow p))$, then $\Box(\text{the Lewis story} \rightarrow p)$. It is also validated on the following subjunctive explication of the story prefix: $Wp \leftrightarrow$ if the Lewis story were true, p would be true. For if (if the Lewis story were true, then if the Lewis story were true, then p would be true), then (if the Lewis story were true, then p would be true).

The proof of (T1) runs as follows. Suppose for reductio that $W(\sim p \ \& \ Wp)$. Then, by (A), $W\sim p$ & WWp . The right conjunct, by (B), entails Wp . This, by (C), entails $\sim W\sim p$. So, we derive: $W\sim p$ & $\sim W\sim p$. Contradiction. Rejecting our assumption, by reductio, gives us $\sim W(\sim p \ \& \ Wp)$. QED.

Where p is an unrestricted locution (e.g. ‘there is a plurality of worlds’), a Fitch-like proof of (T2) may be developed as follows:²⁰

- | | | |
|-----|--|---------------------------|
| (1) | $\forall p(p \rightarrow Wp^*)$ | from (Fic), left to right |
| (2) | $\diamond(\sim p \ \& \ Wp) \rightarrow W(\sim p \ \& \ Wp)$ | from 1, Divers |
| (3) | $\sim W(\sim p \ \& \ Wp)$ | instance of T1 |
| (4) | $\sim \diamond(\sim p \ \& \ Wp)$ | from 2, 3 |
| (5) | $\Box \sim(\sim p \ \& \ Wp)$ | from 4, D |
| (6) | $\Box(Wp \rightarrow p)$ | from 5 |

We suppose at (1) that, for any unrestricted modal or non-modal locution p , if p is true, then according to the Lewis story, the translation of p into the language of counterpart theory obtains. (2) substitutes the unrestricted possibility claim, $\diamond(\sim p \ \& \ Wp)$, for p in (1). Following Divers’ realist translation schema for unrestricted claims, $\diamond p \dashv\vdash p$, the modal realist translates $\diamond(\sim p \ \& \ Wp)$ as $\sim p \ \& \ Wp$, and the fictionalist translates it as $W(\sim p \ \& \ Wp)$. Line (3) is an instance of the theorem, (T1). Line (4) follows trivially from lines (2) and (3). By (D), we derive line (5) from line (4). In classical logic line (6) is equivalent to line (5). Therefore, if fictionalism is true, then

²⁰ In line 2 the star drops off as the result of applying Diver’s translation scheme for unrestricted locutions to an unrestricted locution. Thanks to an anonymous referee here.

every unrestricted claim that is true according to the Lewis story is true: $T2 \ p \rightarrow Wp^* \vdash \Box(Wp \rightarrow p)$.²¹

According to the Lewis story, there is a plurality of worlds. By T2 and (Fic), we can derive that there is a plurality of worlds. This, of course, should not be acceptable to the fictionalist.

IV. Keeping Realism at Bay

How will the fictionalist respond? Well, there is, of course, always the option of denying our initial logical resources. I will not rule out that this can be done in a principled manner.

Assuming, however, that no such route is available to the fictionalist, how can he avoid paradox? In this section I will look at four of the main strategies that have been employed in order to avoid Fitch's original result, and try to determine whether the fictionalist can avail himself of similar strategies.

The Intuitionistic Strategy: Since Fitch's proof is classically, but not intuitionistically, valid, the paradox can be avoided by rejecting classical logic.²² Like Fitch's proof, the proof of T2 is classically, but not intuitionistically, valid. Without classical logic we cannot derive line (6) from line (5). An intuitionist, however, is committed to $Wp \rightarrow \sim\sim p$. This entails: $\sim p \rightarrow \sim Wp$. But the latter is evidently absurd from a fictionalist stance. It reads: if it is not the case that p , then it is not the case that according to the Lewis story, p . So, if it fails to be true that there is a

²¹ Divers and Hagen ("The Modal Fictionalist Predicament") think Divers' solution to the problem of unrestricted claims is undermined because the T-theorem (i.e., truth entails possibility) holds for unrestricted claims. But this assumption is controversial. Lewis explicitly denies it. See "Counterpart Theory and Quantified Modal Logic", p. 39-40. My argument does not rest on this assumption. Of course, it may be thought that if we deny the T-theorem for unrestricted claims, then the problem of possible unrestricted claims does not arise. But this is not so. For the fictionalist wants to say that unrestricted claims are false in spite of being possible. So, for them, the T theorem does not play a role in arguing for the possibility of an unrestricted claim.

²² For discussion of the intuitionistic strategy see e.g. T. Williamson, "Intuitionism Disproved?" *Analysis* 42 (1982), 203-207, *Knowledge and its Limits*, Oxford University Press, Chapter 12, and C. Wright, (1993 [1987]). *Realism, Meaning and Truth*, 2nd ed., Blackwell.

plurality of worlds, as the fictionalist claims, then it is not the case that according to the Lewis story, there is a plurality of worlds.²³

The Modal Fallacies Strategy: Another important strategy that has provoked significant and interesting debate is that offered by Jon Kvanvig.²⁴ Kvanvig argues that Fitch's result is invalid, owing to a fallacious substitution into a modal context. The problem, Kvanvig says, is that Kp is implicitly quantified. Explicitly it reads $\exists x \exists t (Kxpt)$, which says that there is someone x and a time t such that x knows at t that p . But on Kvanvig's neo-Russellian account of quantified expressions, quantified expressions cannot, in general, be legitimately substituted into modal contexts, hence, the failure of the substitution of the Fitch conjunction, $p \ \& \ \sim Kp$, into the knowability principle. Kvanvig's solution has been criticized on various fronts.²⁵ However, even if it succeeds, the fictionalist cannot avail himself of Kvanvig's strategy. For the whole purpose of strong modal fictionalism is to be able to analyze the sentences of *quantified* modal logic without the usual commitment to possible worlds.

A closely related but equally unsuccessful strategy is this. For the Fitch-like proof to go through it is crucial that $\diamond(\sim p \ \& \ Wp)$ is an unrestricted possibility claim. So, might not the fictionalist simply deny that $\diamond(\sim p \ \& \ Wp)$ can be read as an unrestricted possibility claim? Unfortunately, this is not an option. For the fictionalist is prepared to say that it is possible that both \sim (there is a plurality of worlds), and according to the Lewis story, there is a plurality of worlds. That is, he is prepared to say: $\diamond(\sim p \ \& \ Wp)$. However, on a restricted reading, $\diamond(\sim p \ \& \ Wp)$ cashes out to the obviously false claim: 'For some world w , w does not have a plurality of

²³ Parallel reasons have been given for rejecting the intuitionist solution to Fitch's paradox. See e.g. P. Percival, "Fitch and Intuitionistic Knowability" *Analysis* 50 (1990), 182-187.

²⁴ J. Kvanvig, "The Knowability Paradox and the Prospects for Anti-Realism" *Noûs* 29 (1995), 481-499, and *The Knowability Paradox*, Oxford University Press (forthcoming).

²⁵ See e.g. Williamson, *Knowledge and its Limits*, Chapter 12, Brogaard and Salerno, "Knowability, Possibility and Paradox", forthcoming in D. Pritchard, and V. Hendrix (eds.), *New Waves in Epistemology*, and C. Jenkins, "The Mystery of the Disappearing Diamond", forthcoming.

worlds as part, but according to the Lewis story, it does'. Thus, the substitutional fallacy move is unsuccessful.

The Rigidifier Strategy: A third strategy that has provoked significant debate is that of Dorothy Edgington.²⁶ Edgington's strategy is to bypass the knowability principle altogether. Instead, she requires of knowability the less general thesis:

$$(AKP) Ap \rightarrow \Diamond KAp$$

where 'Ap' is to be read 'it is actually the case that p'. Since $KA(p \ \& \ \sim Kp)$ is not provably inconsistent, this strategy avoids paradox. The fictionalist might, similarly, propose to replace fictionalism with the following weaker thesis:

$$(FicA) Ap \leftrightarrow WAp^*$$

There is, however, little reason to think the fictionalist would want to do that. For if he accepts $p \vdash Ap$, and $WAp \vdash Wp$, which we can reasonably expect, then (FicA) entails $p \rightarrow Wp^*$. This is all we need to get the Fitch-like proof going.

The Restriction Strategy: A more recent and widely discussed strategy to block Fitch's original paradox is to restrict the universal quantifier in 'all truths are knowable'. Neil Tennant, for instance, favors what he calls the 'Cartesian' restriction.²⁷ A proposition 'p' is Cartesian just in case 'Kp' is not provably inconsistent. Tennant's Cartesian knowability principle may be stated thus: all Cartesian truths are knowable.

$$(CKP) p \rightarrow \Diamond Kp, \text{ where } p \text{ is Cartesian.}$$

²⁶ D. Edgington, "The Paradox of Knowability" *Mind* 94 (1985), 557-568.

²⁷ N. Tennant. *The Taming of the True*, Oxford: Clarendon Press (1997), p. 274. A related proposal can be found in M. Dummett, "Victor's Error" *Analysis* 61 (2001), 1-2.

It should be apparent that the Cartesian restriction blocks Fitch's paradox, since Fitch's result requires the substitution ' $p \ \& \ \sim Kp$ ' for ' p ' in ' $p \rightarrow \Diamond Kp$ '. ' $p \ \& \ \sim Kp$ ' is not Cartesian, as $K(p \ \& \ \sim Kp)$ is logically impossible.

The fictionalist's best option may be to follow Tennant's lead and reformulate the fictionalist principle as follows:

(Fic*) $p \leftrightarrow Wp^*$, where Wp^* is not provably inconsistent

Provided that Wp^* is provably inconsistent only when p is an unrestricted claim, we can offer the following more effective formulation of fictionalism.

(Fic**) $p \leftrightarrow Wp^*$, where the quantifiers in p are restricted to worlds.

We, furthermore, propose that the fictionalist treat the unrestricted modalities as primitive S5 modalities.²⁸ On a non-redundancy interpretation of the unrestricted modalities, the possibility of modal realism does not entail its truths. The fictionalist can thus coherently deny 'there is a plurality of worlds', but assent to its possibility.

But there is a potential danger in relying upon restriction to avoid paradox. One major obstacle to Tennant's restriction strategy, for example, is that there are other Fitch-like paradoxes that are not averted by the restriction.²⁹ Tennant has subsequently promised to develop his restriction strategy to protect against these further paradoxes. Whether the fictionalist who restricts is susceptible to similar criticism remains to be seen. If he is, there is then the option of following Tennant's lead and developing more suitable restrictions.

²⁸ This strategy is entirely motivated. First, if Hale is right, then the fictionalist cannot offer an adequate and comprehensive analysis of all possibility claims in non-modal terms. The fictionalist will need to admit primitive modality in order to account for the meaning of the story prefix. Second, the modal realist, too, must acknowledge two kinds of modality. The modal realist analyzes the restricted modal claims of quantified modal logic in terms of possible worlds, but must, if Divers is right, treat unrestricted modalities as redundant.

²⁹ See e.g. Williamson, "Tennant on Knowable Truth" *Ratio* 13 (2000), 99-114, Brogaard, and Salerno, "Clues to the Paradoxes of Knowability: Reply to Dummett and Tennant" *Analysis* 62 (2002), 143-150, Brogaard and Salerno, "Knowability, Possibility and Paradox", and Brogaard and Salerno, "Knowability and the Closure Principle", *American Philosophical Quarterly* 43 (2006): 261-270.

But there is a further worry about restriction strategies. Against Tennant's strategy it has been argued that the restriction on knowable truth is unprincipled---that no reason has been given, other than the threat of paradox, to restrict the knowability principle.³⁰ A related charge against Tennant's restriction strategy is that we must admit that however plausible the knowability principle is for a restricted class of sentences, it is to be rejected as a general principle. This is a stern confession on the part of the semantic anti-realist, who claims to have on offer an epistemic *theory* of truth.

A related charge can be issued against the proposed restriction of fictionalism, i.e. (Fic**). Any re-interpretation of modal discourse must be inferentially adequate. The reinterpretation of modal discourse, for example, must inherit the inferential advantages of using discourse about possible worlds.³¹ However, the proposed reinterpretation of modal discourse does not inherit the inferential advantages of possible world semantics. For the modal realist can account for the validity of our standard modal inferences. For example, where p is world-restricted, the modal realist can account for the validity of $p \vdash \Diamond p$ by translating p into an idiom of counterpart theory, inferring the first-order consequence that there is a world in which p , and then translating this consequence back into an ordinary modal idiom. Since the unrestricted modalities are redundant, validity is even easier to account for when p is unrestricted. By contrast, the fictionalist can account only for the validity of $p \vdash \Diamond p$, where p is world-restricted. Hence, fictionalism does not inherit the inferential advantages of using discourse about possible worlds without the ontological costs.

We have been assuming strong modal fictionalism. Strong modal fictionalism holds that the fiction of possible worlds provides the resources for an analysis of modal claims, and so that

³⁰ See for instance, M. Hand and Kvanvig, "Tennant on Knowability" *Australasian Journal of Philosophy* 77 (1999), 422-428, D. DeVidi and T. Kenyon, "Analogues of Knowability" *Australasian Journal of Philosophy* 81 (2003), 481-495, and Hand, "Knowability and Epistemic Truth" *Australasian Journal of Philosophy* 81 (2003), 216-228.

³¹ Rosen, "Modal Fictionalism", p. 330, and Divers, "Agnosticism About Other Worlds: A New Antirealist Programme in Modality", *Philosophy and Phenomenological Research* LXIX (2004), 660-85, p. 665.

modal claims depend on the content of the fiction of possible worlds. Some of the problems with fictionalism can be avoided if, as Rosen suggests, we take fictionalism to provide ‘not a theory of possibility, but merely a theory linking the modal facts with facts about the story PW’.³² The result is timid modal fictionalism.³³ Just like strong modal fictionalism, timid modal fictionalism licenses only a subset of transitions from modal idioms to idioms of counterpart theory. However, this may be unproblematic insofar as the timid fictionalist does not regard (Fic**) as purporting to shed light on the nature of modal truth, but merely take it to link the sentences of quantified modal logic with the sentences of counterpart theory.

V. General Lessons

Of the four solutions to Fitch’s result considered, only restriction strategies can be extrapolated to block the Fitch-like result developed above. This shows that restriction strategies are potentially more effective tools for avoiding Fitch-like paradoxes than are the other strategies we have considered. Unless it can be argued that different anti-realisms call for fundamentally different resolutions of Fitch-like paradoxes,³⁴ the aptitude of restriction strategies to block Fitch-like paradoxes should count in their favor.

The downside is that the restriction strategist cannot take the principles he is restricting as providing the resources for an analysis of truth. The result of restriction is thus timid anti-realism. Is timid semantic anti-realism an acceptable position? Well, one of the motivations for

³² “Modal Fictionalism”, p. 354.

³³ For a defense of timid modal fictionalism, see Brogaard, “Two Modal -Isms: Fictionalism and Ersatzism”, forthcoming in Hawthorne, ed. *Philosophical Perspectives*.

³⁴ Fitchy paradoxes may be symptomatic of a common malady in (a range of) antirealist positions. It would be interesting to find out what the malady is. Is there a deep metaphysical point to be made about why these different positions give rise to this distinctive kind of paradox? On the face of things, it is not easy to see what that might be. Anti-realism and modal fictionalism both consist in the non-acceptance of the objectivity of a certain subject matter (e.g. modality), but apart from this superficial similarity, they are obviously very different. In a recent paper [Brogaard and Salerno, “Anti-Realism, Theism, and the Conditional Fallacy”, *Nous* 39 (2005)], it was argued that the malady in question is a kind of conditional fallacy. Perhaps then, it is not that modal fictionalism is similar in some deeper respect to semantic anti-realism, but rather that strong anti-realisms run into some kind of conditional fallacy. Thanks to John Divers here.

semantic anti-realism is Dummett's manifestation argument.³⁵ The argument is often taken to be that, for reasons having to do with the manifestability of meaning, truth is to be understood epistemically, in terms of what our epistemic capacities allow us to verify in principle. But if it can be shown that manifestability considerations entail no such result, then it is open to argue for a timid semantic anti-realism. A timid semantic anti-realist could appeal to verifiability merely in order to counter the realist claim that even truths that can be consistently known may be beyond our epistemic reach.³⁶

In conclusion: the present case study indicates that Fitch-like paradoxes present a major obstacle, not only to semantic anti-realism, but also potentially to a number of other anti-realisms. The Fitch-like paradoxes give anti-realists reason to restrict. Restriction leads to timid anti-realism. Fitch's proof can thus be construed as an argument for timid (as opposed to strong) anti-realism.³⁷

³⁵ Dummett, "What is a Theory of Meaning? (II)" in G. Evans and J. McDowell (eds.), *Truth and Meaning*, (Clarendon Press, 1976), Chapter 4, *Elements of Intuitionism*, (Oxford: Oxford University Press, 1977), "The Philosophical Basis of Intuitionistic Logic" in Dummett (ed.), *Truth and Other Enigmas*, (Harvard University Press, 1978).

³⁶ This, in fact, may be all that Tennant is committed to. See Tennant, "Is Every Truth Knowledge? Reply to Hand and Kvanvig", *Australasian Journal of Philosophy* 79 (2001), 107-13.

³⁷ I am indebted to Joe Salerno for invaluable discussion, and to Joe, John Divers, Michael Hand, David Jehle, Julien Murzi and an anonymous referee for Oxford University Press for helpful comments.